Fitting the relic density with dimension-five contributions

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Abstract. We investigate the behaviour of the relic density for a heavy dark matter model with a dimension-five operator added to the theory. The model features a top-philic dark matter candidate and heavy fermionic mediator which interact via a Yukawa-type term, and which in mass-degenerate set-ups results in coannihilation effects on the relic density. We present a semi-analytical fit to the relic density, modelling the interplay between the dimension-five contact term and Yukawa-type contribution, and showing that coannihilation effects should not be neglected in a naive fit. Additionally, we motivate the use of a semi-analytic fit in place of computer-intensive simulations, showing that the functional form is able to predict parameters producing the correct relic density.

1. Introduction

The dark matter puzzle is one of the foremost questions in particle physics today, and one which forms a significant part of many experimental programs. It has been well established that about 25 % of the energy in the universe is taken up by dark matter [1], so-named for its non-interaction with light [2], but its nature remains unknown. If it is a particle, it does not form part of the Standard Model (SM). Physics programmes designed to detect it include astrophysical attempts using indirect and direct detection, and experiments at colliders such as the Large Hadron Collider at CERN. None of these efforts have yet yielded conclusive discoveries.

In the following, we investigate the phenomenology of a heavy scalar dark matter candidate S which is top-philic and couples to the SM via a heavy fermionic mediator, T, via a t-channel interaction. The new physics states S and T are both odd under a Z_2 symmetry, and the masses of both resonances are constrained to lie in the range 200 GeV $\leq m_S, m_T \leq$ 3 TeV. The upper limit is chosen to match the scale of a generic composite Higgs theory, which is expected to be several TeV. We build upon a previous model studied in Refs. [3, 4], where it was shown that the next-to-leading-order (NLO) contributions to the annihilation cross section should not be neglected for heavy dark matter. Furthermore, it is a goal of this work to postulate that S and T

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arise as bound states in a composite Higgs model featuring underlying fermions, and to support this we enforce that their masses lie within one order of magnitude. To further investigate this avenue, we follow an effective theory approach and add to the existing model [3, 4] an additional contact interaction $SStt$ with an unknown $\mathcal{O}(1)$ coefficient. Such a dimension-five term is a generic feature of a broad range of Beyond the Standard Model theories, including composite Higgs models, arising from strong dynamics. Following Refs. [3, 4], we add to the SM the minimal Lagrangian

$$\mathcal{L} = i\bar{T}\not{D}T - m_T\bar{T}T + \frac{1}{2}\partial_\mu S\partial^\mu S - \frac{1}{2}m_S^2 S^2 + \left[\tilde{y}_t S\bar{T}P_R t + h.c.\right] + \frac{C}{\Lambda}SSt\bar{t}, \tag{1}$$

where m_T is the mass of the mediator and m_S is the mass of the dark matter candidate. The dark matter interaction with the SM sector is achieved through the Yukawa-type operator with coefficient $\tilde{y}_t$ and the dimension-five term with coefficient C/Λ. The parameter Λ indicates the energy scale of the effective theory, such as the range of validity of compositeness. When the masses of S and T are near to the compositeness scale, the validity of the EFT should be treated with care. However, colliders such as the LHC probe scales of 1 TeV or less in typical dark matter searches, implying these predictions may be safely trusted there.

A dark matter candidate S in the presence of mediator T with a small mass splitting between the two leads to coannihilations becoming important in the calculation of the relic density. In this case, the relic abundance of dark matter is not only controlled by the process $\sigma(SS \to \text{SM SM})$, but also by $\sigma(ST \to \text{SM SM})$. The rate of annihilation of dark matter may then also be indirectly impacted by annihilations of the mediator, $\sigma(TT \to \text{SM SM})$.

The thermally averaged cross section $\langle\sigma v\rangle$ may be non-relativistically expanded as

$$\langle\sigma v\rangle \approx a + b\langle v^2\rangle + \mathcal{O}(v^4), \tag{2}$$

with v the relative velocity between the two scattering dark matter candidates. The annihilation rate features contributions from partial waves of the scattering amplitude, where the term a on the right hand side of equation (2) corresponds to the velocity-independent s-wave term, and the second represents the p-wave contribution, and scales with v^2. In simple dark matter models, the s-wave contribution to the annihilation cross section dominates, and higher partial waves are minimal [5].

In a phenomenon dubbed the WIMP miracle, a weak-scale particle generically undergoes thermal freeze-out within a few orders of magnitude of the correct cross section, making the measured relic density $\Omega_{DM}h^2 = 0.1186 \pm 0.0020$ [6] easily achievable. In the case without coannihilations, the relic density of S, $\Omega_{DM}h^2$ [7], is the solution to the Boltzmann equation [7]

$$\frac{dn}{dt} = -3Hn - \langle\sigma_{eff}v\rangle\left(n^2 - n_{eq}^2\right), \quad \Omega_{DM}h^2 \approx \frac{1.04\times 10^9}{M_{Pl}}\frac{x_F}{\sqrt{g_*(x_F)}}\frac{1}{a+3b/x_F} \tag{3}$$

where H is the Hubble constant, the number density of S is given by n, the equilibrium number density is indicated by n_{eq}, M_{Pl} is the Planck mass, g^* the number of effectively massless degrees of freedom, and $x = m_S/T_x$, with T_x the freeze-out temperature. In the case at hand, the NLO annihilation cross section for SS annihilation is a sum of tree level and virtual internal bremsstrahlung contributions[1] [4]

$$\sigma v_{NLO} \approx \sigma v_{q\bar{q}} + \sigma v_{VIB}^{(0)}, \qquad \text{with } \langle\sigma v\rangle = \sigma v_{NLO} + \sigma v_{SStt}. \tag{4}$$

[1] For the full expression of the NLO cross section, the interested reader is referred to Ref. [4], and for the dimension-five cross section to Ref. [8].

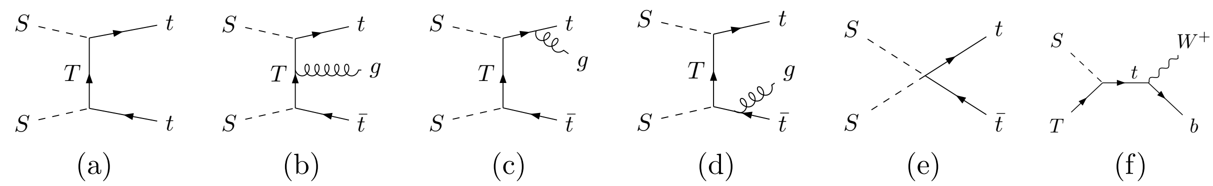

Figure 1. Relevant Feynman diagrams for the annihilation and co-annihilation (with T) of S.

From equation (3), we use that $\Omega_{DM}h^2 \sim 1/\langle\sigma v\rangle$ [9], solving for C/Λ yields

$$\frac{C}{\Lambda} = f(m_S, m_T, \tilde{y}_T) \approx \frac{1}{\sqrt{A(m_S)}}\sqrt{b'(x_F, g_*(x_F)) - B(m_S, m_T)\ \tilde{y}_t^4}, \tag{5}$$

with[2]

$$\begin{aligned} A(m_S) &= \frac{\Lambda^2\langle\sigma v\rangle_{SStt}}{C^2} = \frac{N_c}{4\pi}\left(1 - \frac{m_t^2}{m_S^2}\right)^{3/2}, \quad B(m_S, m_T) = \frac{\sigma v_{q\bar{q}} + \sigma v_{VIB}^{(0)}}{\tilde{y}_t^4}, \\ b'(x_F, g_*(x_F)) &= \left(7.2 \times 10^{-10}\ \text{GeV}^{-2}\right)\frac{x_F}{\sqrt{g_*(x_F)}}, \end{aligned} \tag{6}$$

where m_t is the mass of the top quark, and $b'(x_F, g_*(x_F))$ is fitted from the numerical result. Here, $A(m_S)$ and $B(m_S, m_T)$ are obtained by factorising the coefficients from each of the relevant cross sections, so as to use those coefficients as variables for the fit.

2. Considering coannihilations

The canonical calculation of dark matter relic density must, however, be modified when coannihilations with another relatively mass-degenerate state are possible. This is highlighted in figure 1, where diagrams (a) through (e) illustrate the SS annihilation channels, diagram (f) presents an example coannihilation channel. In the following, we present the steps for including such features in the calculation of the relic density, before presenting the fit.

Consider the case at hand, where $m_S < m_T$. The abundance of S is determined by a set of Boltzmann equations [10]

$$\frac{dn}{dt} = -3Hn - \langle\sigma_{ij}v\rangle(n^2 - n_{eq}^2), \tag{7}$$

which is of the same form as the Boltzmann equation for the single particle annihilation. It can then be solved by the same techniques, through performing the annihilation integral after solving for the freeze out temperature. More detail on this process is available in Ref. [10]. In that case, the additional state (here, the mediator T), can effect the annihilation cross section. In the general case, the effective thermally averaged cross section due to coannihilations is given by [10]

$$\begin{aligned} \sigma_{eff}(x) &= \sum_{ij}^{N}\sigma_{ij}\frac{g_i g_j}{g_{eff}^2}(1+r_i)^{3/2}(1+r_j)^{3/2}\exp(-x(r_i+r_j)), \\ g_{eff}(x) &= \sum_{i=1}^{N} g_i(1+r_i)^{3/2}\exp(-x\ r_i), \qquad \text{with } r_i = \frac{m_i}{m_S} - 1, \end{aligned} \tag{8}$$

[2] For the full expression of $B(m_S, m_T)$ and further details of the calculation, the interested reader is referred to Ref. [8].

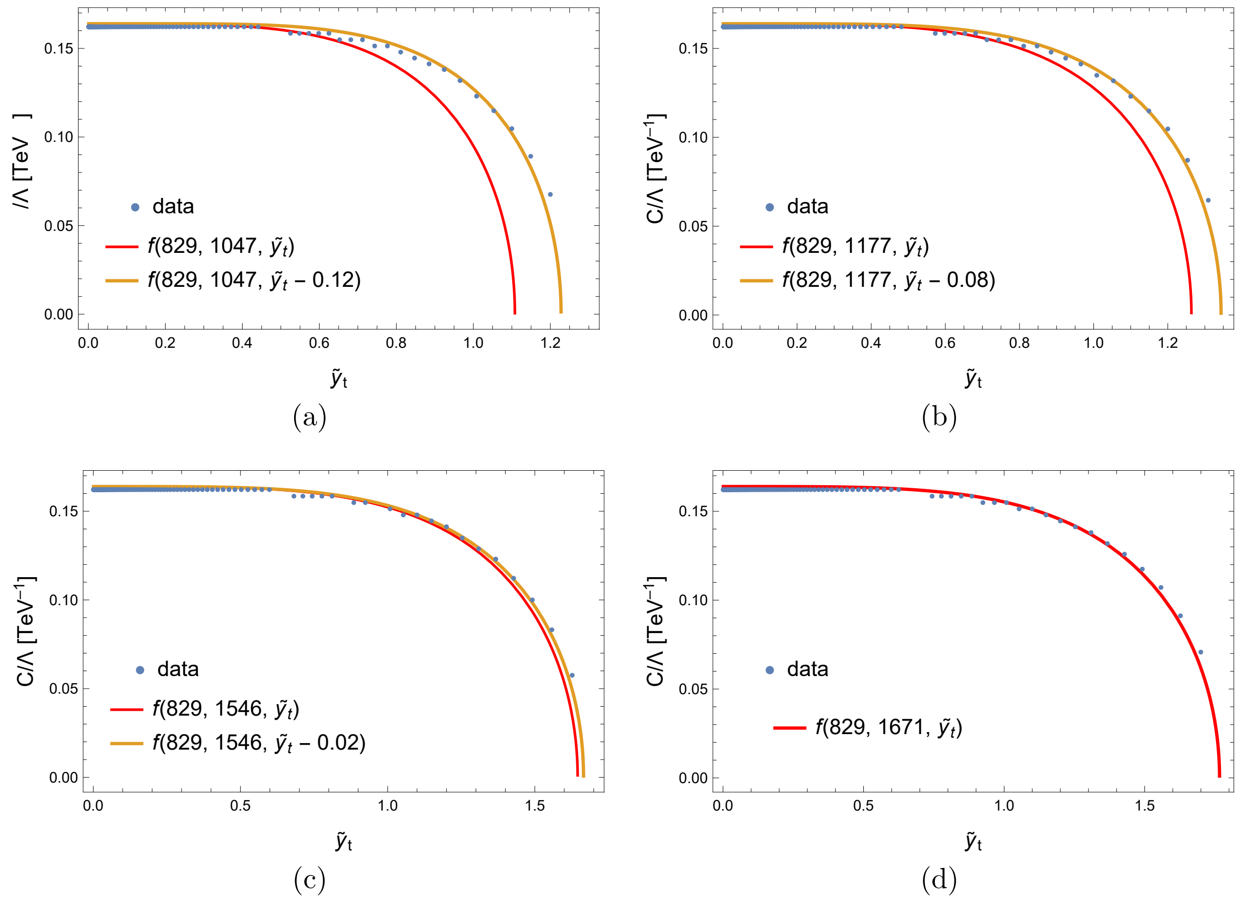

Figure 2. The shift due to coannihilations for $m_S = 829$ GeV, with mass ratios $r = 0.26$, $r = 0.42$, $r = 0.86$, and $r = 1.01$ for plots $(a - d)$. All points yield the correct relic density.

recalling that $x = m_S/T_x$. Here, $\sigma_{ij} \equiv \sigma(\chi_i\chi_j \to \text{SM})$ for states χ_i with g_i internal degrees of freedom. In this case, with only two states $S = \chi_i$ and $T = \chi_j$ relevant, the effective annihilation cross section simplifies to

$$\sigma_{eff}(x) = \sigma_{SS} + \sigma_{ST}\,\frac{g_S g_T}{g_{eff}^2}\left(\frac{m_T}{m_S}\right)^{3/2} \exp\big(-x\,r\big), \tag{9}$$

where $\sigma_{ST} \propto \tilde{y}_t^2$. The form of equation (9) displays an exponential dependence attached to the coannihilation-induced cross section. That is, for smaller values of r, we expect to observe a larger modification to σ_{SS} (where $x > 0$ always). We now move to examining the semi-analytical fit to the relic density, bearing these contributions in mind.

3. Semi-analytical fit

The dark matter relic density is simulated using the MICROMEGAS [11] framework, which allows the user full control of the cross section and numerical calculation of the relic abundance, and includes all relevant annihilation and coannihilation channels. The CalcHEP [12] model file was generated using the FeynRules [13, 14]/CalcHEP interface [13]. In this work, we have improved the estimation of $\langle\sigma v\rangle$ by including the NLO form of the cross section within the framework. The free parameters of the scan are varied with masses m_S $(m_T) \in [200 -$

3000(3500)] GeV, $\tilde{y}_t \in [10^{-4}, 6]$, and $C/\Lambda \in [10^{-3}, 10^{-5}]$ GeV^{-1}. The masses are chosen to lie above the top mass (avoiding threshold effects) and roughly below the envisioned compositeness scale, which typically lies around 3 TeV. The justification for the Yukawa parameter bounds is borrowed from previous investigations [3], where the upper limit is the limit of the perturbative regime, and the lower limit ensures that MicrOmegas correctly handles co-annihilation effects. Finally, to establish the bounds for the contact term coefficient, the values of C/Λ which take over from the Yukawa term ($\tilde{y}_t \sim 0$) in producing the correct relic density were established, as well as the largest value of C/Λ which does not modify the relic density due to the Yukawa term.

In studying the modification of the relic density behaviour for a benchmark at hand, we study the behaviour in the $\tilde{y}_t - -C/\Lambda$ plane, using as benchmark data the numerical predictions from the relic density simulations. In particular, as shown in figure 2 for $m_S = 829$ GeV, we observe that as expected the smaller $r = m_T/m_S - 1$ values deviate from the simple fit of equation (5). The fit has been further detailed in Ref. [8], so here we will simply quote the result. The shift relevant to a mass point (m_S, m_T) which influences the Yukawa-type parameter is found to follow

$$s(m_S, m_T) = \begin{cases} 0.4k^r & m_S \leq 1.2 \text{ TeV} \\ 0.7k^r & m_S > 1.2 \text{ TeV} \end{cases}, \tag{10}$$

where $k(m_S)$ is an unconstrained dimensionless parameter. The change in behaviour at roughly 1 TeV is motivated by the appearance of NLO effects at that energe regime, which is independant of the value of Λ chosen. The value for $k(m_S)$ may be found via a fit to an exponential function

$$k(m_S) = \begin{cases} \left(1.9 \times 10^{-4}\right) \left(6.2 \times 10^{8}\right)^{m_S/\Lambda} & m_S \leq 1.2 \text{ TeV} \\ (3.0 \times 10^{-3})(1.8 \times 10^{4})^{m_S/\Lambda} & m_S > 1.2 \text{ TeV} \end{cases}, \tag{11}$$

for $\Lambda = 3.5$ TeV, which yields the semi-analytic fit result

$$\frac{C}{\Lambda} \approx f(m_S, m_T, \tilde{y}_t) = \frac{1}{\sqrt{A(m_S)}} \sqrt{b' - B(m_S, m_T) \left(\tilde{y}_t - \alpha \left[\beta \gamma^{\frac{m_S}{\Lambda}}\right]^r\right)^4}. \tag{12}$$

The coefficient $b'(x_F, g_*(x_F))$ is determined by the fit as $b' = 6.0 \times 10^{-9} \pm 0.2 \times 10^{-9}$ GeV^{-2}. Additionally, we find the remaining coefficients parametrising the fit to be $(\alpha, \beta, \gamma) = (0.4, 1.9 \times 10^{-4}, 6.2 \times 10^{8})$ for $m_S \leq 1.2$ TeV, and $(\alpha, \beta, \gamma) = (0.7, 3.0 \times 10^{-3}, 1.8 \times 10^{4})$ for $m_S > 1.2$ TeV. In particular, the parameter γ has been raised to a dimensionless ratio, where $\Lambda = 3.5$ TeV is the maximum value for m_T used in the scan. As mentioned above, Λ provides an indicative effective scale, such as the limit of validity of the theory or the scale of compositeness.

4. Applying the shift

The fit to the relic density was performed to motivate its use over time-consuming and CPU-heavy simulations. The function obtained in equation (12) offers an alternative to performing a simulation of the relic density in this context, where the user may instead obtain the values of the $\tilde{y}_t$ and C/Λ parameters which yield the correct relic density for a given benchmark. In order to further motivate this, we apply the functional form (without fitting) to data points to check the agreement. As displayed in figure 3, the functional form closely matches the behaviour of the data, motivating the use of the fit in place of simulation.

In particular, we note good agreement at the 'boundaries' of the parameter space; for large $\tilde{y}_t$ and small C/Λ (and vice versa) the functional form matches the data well. We observe slight deviation from data in the regions where the interplay between the variables is strongest, particularly for larger m_S. This behaviour may be expected generally for t-channel dark matter theories which feature coannihilations due to small mass gaps.

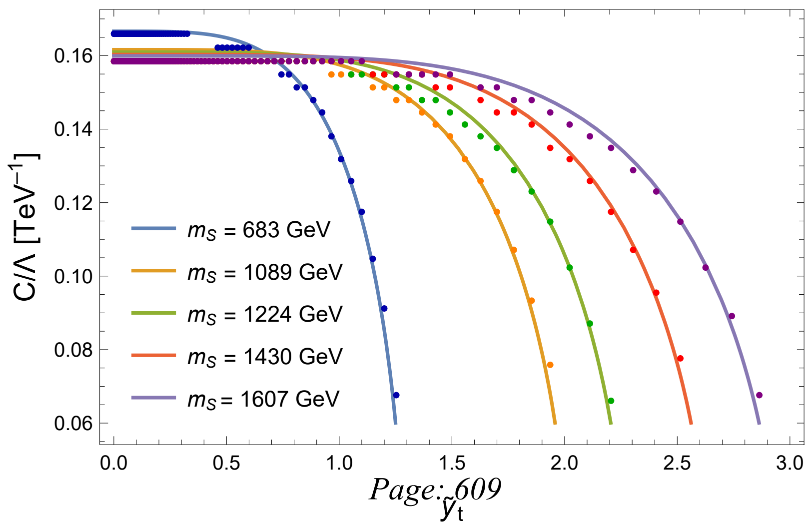

Figure 3. A number of benchmarks with a common mass ratio, $r = 0.72$ are presented, where the shift due to coannihilations is included. All points achieve the correct relic density.

5. Conclusion

This work has demonstrated that the relic density for a simple dark matter model may be modelled through a simple semi-analytical fit, which takes into account coannihilation effects. Additionally, we have motivated the possibility that a functional fit of this nature could conceivably take over from an intensive computer simulation, and may be used to access the parameters yielding the correct relic density for additional benchmarks or mass splittings.

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